Iterative solution of eddy current problems on polyhedral meshes

Paolo Bettini[1], Mauro Passarotto[2], Ruben Specogna[2]
[1]Department of Industrial Engineering (DII), University of Padova, Italy, paolo.bettini@unipd.it
[2]Polytechnic Department of Engineering and Architecture (DPIA), University of Udine, Italy, ruben.specogna@uniud.it

We present an effective technique to solve eddy current problems on conductors of arbitrary topology by an iterative geometric formulation suitable for an arbitrary polyhedral mesh. The formulation is based on the reformulation of the volume integral formulation in a form suitable to be solved by fixed point iterations. Given that nowadays the computation of Biot–Savart fields can be performed efficiently on GPUs, the right hand side of the systems is constructed at each fixed point iteration without computing the inductance matrix which is fully populated. The proposed technique is useful in problems where meshing the complement of conductors is hard, thus representing a sound alternative to integral formulations.

***Index Terms*—eddy currents, discrete geometric approach (DGA), Finite Elements, fixed point, cohomology**

I. Introduction

THIS PAPER addresses the solution of eddy current problems in conducting structures of arbitrary topology by means of a geometric integral formulation suitable for a general polyhedral mesh. The formulation may be derived from the geometric integral formulation introduced in [1], which is similar to the Finite Element counterpart [2]. The main differences are that [1] is suitable with general polyhedral meshes and the topological pre-processing is performed much faster thanks to state of the art algorithms as the Dłotko–Specogna (DS) [3].

Assuming, for the sake of simplicity, that the conductor is topologically trivial, either of the formulations [1] or [2] may be written as a linear system

$$(\mathbf{K}_R + i\,\mathbf{K}_M)\mathbf{T} = \mathbf{b}_s, \tag{1}$$

where $\mathbf{K}_R$ and $\mathbf{K}_M$ are the real and imaginary part of the system matrix, respectively and $\mathbf{b}_s$ is the right-hand side. Finally, $\mathbf{T}$ is the vector of unknowns that can be interpreted as the integral of the electric vector potential on mesh edges. We remark that $\mathbf{K}_M$ is fully populated, while $\mathbf{K}_R$ is very sparse.

The main idea, inspired from [4], [5], stems from rewriting the linear system as

$$\mathbf{K}_R\,\mathbf{T} = -i\,\mathbf{K}_M\,\mathbf{T} + \mathbf{b}_s, \tag{2}$$

which yields

$$\mathbf{T} = -i\,\mathbf{K}_R^{-1}\,\mathbf{K}_M\,\mathbf{T} + \mathbf{K}_R^{-1}\,\mathbf{b}_s. \tag{3}$$

Then, (3) is solved by a fixed point iteration, i.e.

$$\mathbf{T}^n = -i\,\mathbf{K}_R^{-1}\,\mathbf{K}_M\,\mathbf{T}^{n-1} + \mathbf{K}_R^{-1}\,\mathbf{b}_s. \tag{4}$$

We remark that, similarly to the T-method [4], the matrix $\mathbf{K}_M$ still has to be assembled. Since this matrix is full, it requires too much memory to be stored in a computer for large problems. The novelty of this contribution is twofold: how to introduce an alternative technique that avoids the computation of matrix $\mathbf{K}_M$ and how to deal with non simply connected conductors.

Instead of computing $\mathbf{K}_M$, it is in fact advantageous to compute the right-hand side of (3) at each fixed point step by Biot–Savart law. This can be performed very quickly nowadays by performing the computations on a GPU. Applications where the proposed formulation is most useful are the ones for which the generation of the mesh in the complements of conductors may be problematic. For example, in nuclear fusion applications, where the complexity of the conducting structures surrounding the plasma can prevent the generation of the mesh in air, as required by standard differential formulations. For this reason, these problems are often solved by means of integral formulations. Nonetheless, a fine discretisation of the conductors require the adoption of advanced techniques (e.g. fast multipole method or adaptive cross approximation coupled with hierarchical matrix arithmetics) to avoid impractical memory requirements. The proposed formulation is a simple and sound alternative. Next section contains the details of the novel formulation.

II. The novel formulation

The whole algorithm is represented in Fig. 1. Before starting the cycle, the arrays $\mathbf{b}_s$ is computed. In our geometric formulation this is obtained with

$$\mathbf{b}_s = -i\omega\,\mathbf{C}^T\,\tilde{\mathbf{A}}_s, \tag{5}$$

where ω is the angular frequency, $\mathbf{C}$ is the sparse matrix containing the incidences between the face and edge pairs and $\tilde{\mathbf{A}}_s$ is the integral of the magnetic vector potential (computed by using the Biot–Savart law) on dual edges of the mesh due to the source currents only.

Then, a cycle is performed until the current update $|\mathbf{I}^n - \mathbf{I}^{n-1}| = |\mathbf{C}^T\,\mathbf{T}^n - \mathbf{C}^T\,\mathbf{T}^{n-1}|$ is below a user-defined tolerance. In each iterate, the old current density inside mesh elements is found by multiplying the face basis functions [7] by the current $\mathbf{I}^{n-1} = \mathbf{C}^T\,\mathbf{T}^{n-1}$ on mesh faces. Then, the magnetic flux $\tilde{\mathbf{\Phi}}_c^{n-1}$ on dual faces produced by eddy currents is computed as

$$\tilde{\mathbf{\Phi}}_c^{n-1} = \mathbf{C}^T\,\tilde{\mathbf{A}}, \tag{6}$$

where the magnetic vector potential integrated on dual edges $\tilde{\mathbf{A}}$ is found with Biot–Savart law from the induced current density. Finally, the new distribution of current is found by solving a linear system

$$\mathbf{K}_R\mathbf{T}^n = -i\omega\tilde{\mathbf{\Phi}}_c^{n-1} + \mathbf{b}_s \tag{7}$$

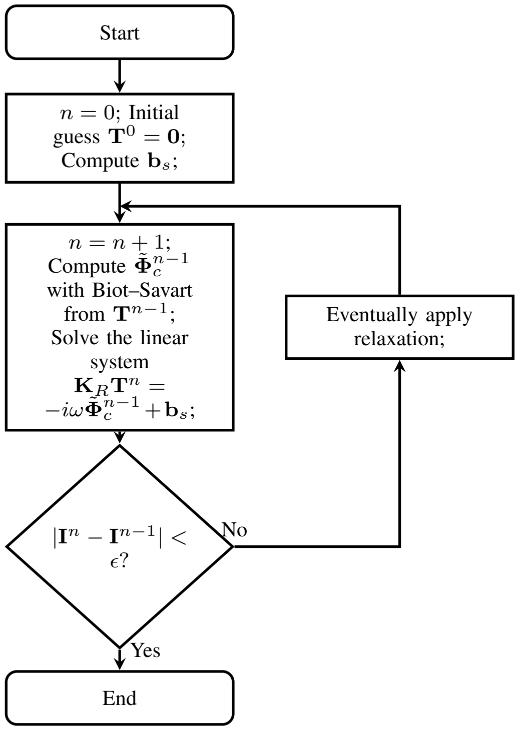

Fig. 1. Iterative solution of an eddy current problem.

which enforces the discrete Faraday's law. We remark that one may use any div-conforming method to compute such current. That is, one may use the gauged or ungauged electric vector potential formulation. In our implementation we use the faster mixed-hybrid formulation described in [6], which produces a system that is amenable to algebraic multigrid and does not require to deal with issues related to the topology of the conductors. Yet, the results produced by using all the mentioned formulations is exactly the same up to solver tolerance given that the formulations are algebraically equivalent.

The proposed algorithm works also for non-simply connected conductors,provided that cohomology basis functions are used inside the electric vector potential formulation. In the full paper, we are going to show that using the mixed-hybrid formulation avoids any issues related to the topology of the conductors in such a way that computing a cohomology basis is no longer required.

III. Numerical results

The proposed approach has been applied to calculate the currents induced in a solid sphere (radius $r=50mm$, resistivity $\rho=0.1\mu\Omega m$) subject to a uniform sinusoidal magnetic field ($B=1T$, $f=50Hz$). The numerical domain is covered by a polyhedral mesh consisting of 1840 elements, 7256 faces, 7326 edges and 1911 nodes. The total number of DoFs is 4329. The solution, shown in Fig. 2, is in excellent agreement with the one obtained with the volume integral formulation presented in [1]. The full paper will contain a more extensive and quantitative comparison between the two methods.

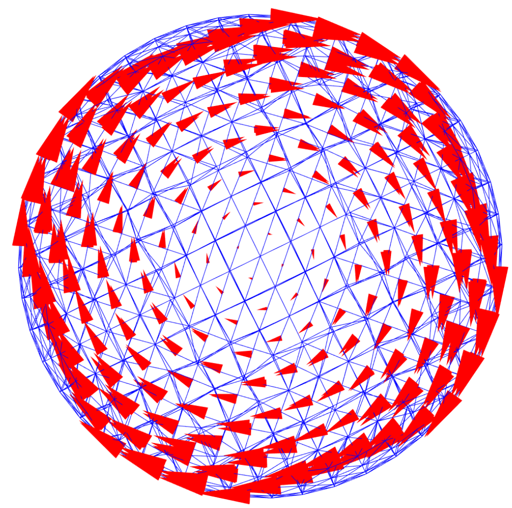

Fig. 2. Eddy currents induced in a solid sphere (radius $r=50mm$, resistivity $\rho=0.1\mu\Omega m$) subject to a uniform sinusoidal magnetic field ($B=1T$, $f=50Hz$) in the direction perpendicular to the page. Red cones: imaginary part of the current density.

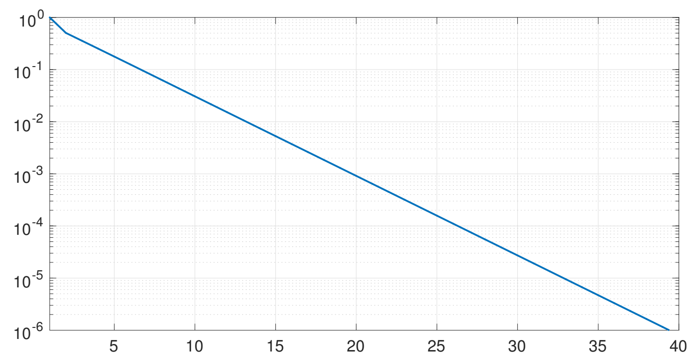

Fig. 3. Convergence of the method: absolute error $\varepsilon = |\mathbf{I}^n - \mathbf{I}^{n-1}|$ as a function of the number of iterations n.

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